# Interactive Options Hedging ILP

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# What is an Option?

An option is a financial contract giving the holder the right (but not the obligation) to buy or sell an asset at a predetermined price.

Two primary types:

- **Call option**: right to buy
- **Put option**: right to sell

In this presentation, we focus entirely on S&P-linked options.

# Strike Price and Maturity

Key option terminology:

- **Strike price**: the fixed price at which the option can be exercised
- **Maturity**: the expiration date of the option

Examples:

- SPX 4200 Put, 3M maturity
- SPX 4800 Call, 12M maturity

Different strikes and maturities produce different payoff profiles under market moves.

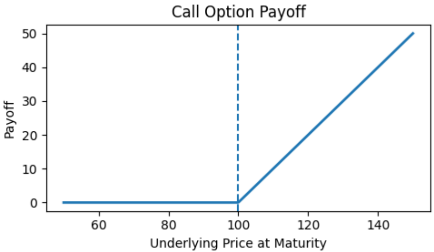
# Call Option Payoff

A call option benefits when the underlying asset rises above the strike price.

$$\text{Call Payoff} = \max(S_T - K, 0)$$

**Variables**

- $S_T$: underlying asset price at maturity
- $K$: strike price



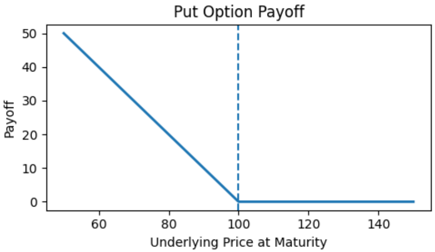
# Put Option Payoff

A put option benefits when the underlying asset falls below the strike price.

$$\text{Put Payoff} = \max(K - S_T, 0)$$

**Variables**

- $S_T$: underlying asset price at maturity
- $K$: strike price



# Why Use Options?

Common use cases for options:

- Speculation on market direction
- Leverage and capital efficiency
- Income generation
- Volatility trading
- **Hedging risk**

This presentation focuses on using S&P options to hedge portfolio exposure under stress scenarios.

# Disclaimer

**Important Disclaimer**

- All funds, option contracts, market scenarios, PnL values, and constraints shown in this presentation are entirely fictional.
- All numerical inputs are fabricated for illustrative purposes only.
- The data is *not* intended to reflect:
  - actual market pricing,
  - real liquidity,
  - actual option availability,
  - realistic implied volatilities,
  - or any real trading recommendation.
- This deck is solely intended to explain the ILP formulation and optimization workflow.

# Problem Motivation

Goal:

- Hedge multiple portfolios using only S&P options
- Improve downside protection under stress scenarios
- Minimize trading activity
- Keep implementation operationally simple

# Sample Funds

| Fund | Initial Aggregate PnL |
|---|---:|
| Plum | \$12.4MM |
| Pear | \$8.7MM |
| Peach | \$15.9MM |

# Market Scenarios

S&P stress scenarios:

$$
\begin{aligned}
&-20\%, -10\%, -5\%, \\
&+5\%, +10\%, +20\%
\end{aligned}
$$

# S&P Options Universe

| Option | Strike | Maturity |
|---|---|---|
| SPX 4200 Put | 4200 | 3M |
| SPX 4300 Put | 4300 | 6M |
| SPX 4400 Put | 4400 | 9M |
| SPX 4700 Call | 4700 | 6M |
| SPX 4800 Call | 4800 | 12M |
| ⋮ | ⋮ | ⋮ |

# Sample Option Payoff Matrix

| Option | -20% | -10% | -5% | +5% | +10% | +20% |
|---|---|---|---|---|---|---|
| SPX 4200 Put | 420k | 180k | 75k | -40k | -65k | -90k |
| SPX 4300 Put | 510k | 220k | 90k | -50k | -80k | -120k |
| SPX 4400 Put | 620k | 310k | 130k | -60k | -100k | -160k |
| SPX 4700 Call | -120k | -70k | -30k | 95k | 180k | 420k |
| SPX 4800 Call | -90k | -45k | -20k | 110k | 240k | 520k |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |

# Decision Variables

| | |
|---|---|
| $X_{ik} \in \mathbb{Z}$ | Contracts traded |
| $Y_{kj} \in \mathbb{R}$ | Final PnL |
| $A_{ik} \geq 0$ | Absolute trade size |
| $U_i \in \{0, 1\}$ | Whether option $i$ is used |

**Variables**

- $i$: option index
- $k$: fund index
- $j$: scenario index

# Challenge 1: Modeling PnL Impact

Question:

How do we mathematically represent the total PnL contribution from all option trades under a given scenario?

We need:

- Contributions from every option
- Different payoffs under each scenario
- Aggregation across all trades

# Solution: PnL Impact Constraint

$$D_{kj} = \sum_{i \in I} p_{ij} X_{ik}$$

Interpretation:

- $p_{ij}$ = payoff of option $i$ in scenario $j$
- $X_{ik}$ = contracts traded
- Summation aggregates all option impacts

**Variables**

- $D_{kj}$: hedge PnL contribution
- $i$: option index
- $k$: fund index
- $j$: scenario index

# Example PnL Computation

Suppose for Plum under the $-10\%$ scenario:

$$
\begin{aligned}
2 \times (\text{SPX 4300 Put}) &= 2 \times 220k \\
-1 \times (\text{SPX 4700 Call}) &= -1 \times (-70k)
\end{aligned}
$$

Then:

$$D_{kj} = 440k + 70k = 510k$$

**Variables**

- $D_{kj}$: hedge PnL contribution
- $X_{ik}$: contracts traded

# Challenge 2: Defining Final PnL

Question:

How do we combine the existing portfolio exposure with the hedge impact?

# Solution: Final PnL Definition

$$Y_{kj} = a_{kj} + D_{kj}$$

where:

- $a_{kj}$ = initial portfolio PnL
- $D_{kj}$ = hedge impact

**Variables**

- $Y_{kj}$: final PnL
- $a_{kj}$: initial PnL
- $D_{kj}$: hedge impact

# Example Final PnL

Suppose:

$$a_{kj} = -3.9\text{MM}$$
$$D_{kj} = +510k$$

Then:

$$Y_{kj} = -3.39\text{MM}$$

**Variables**

- $Y_{kj}$: final PnL
- $a_{kj}$: initial PnL
- $D_{kj}$: hedge impact

# Challenge 3: Risk Constraints

Question:

How do we ensure that each fund and the overall portfolio remain within acceptable risk limits?

# Solution: Risk Constraints

Fund-level:

$$\ell_j^Y \leq Y_{kj} \leq u_j^Y$$

Portfolio aggregation:

$$T_j = \sum_{k \in K} Y_{kj}$$

Portfolio-level:

$$\ell_j^T \leq T_j \leq u_j^T$$

**Variables**

- $Y_{kj}$: fund-level PnL
- $T_j$: total portfolio PnL
- $\ell_j^Y, u_j^Y$: fund bounds
- $\ell_j^T, u_j^T$: portfolio bounds

# Example Constraint Inputs

Trade bounds:

$$-100 \leq X_{ik} \leq 100$$

Fund-level PnL bounds:

$$-2.0\text{MM} \leq Y_{kj} \leq 4.0\text{MM}$$

Portfolio-level bounds:

$$-1.0\text{MM} \leq T_j \leq 8.0\text{MM}$$

# Challenge 4: Minimizing Trade Size

Question:

We want to minimize total trading volume:

$$|X_{ik}|$$

But absolute values are nonlinear.
How can we convert this into a linear optimization problem?

# Solution: Absolute Value Linearization

Introduce:

$$A_{ik} \geq 0$$

with constraints:

$$A_{ik} \geq X_{ik}$$
$$A_{ik} \geq -X_{ik}$$

Then minimize:

$$\min \sum_{i \in I} \sum_{k \in K} A_{ik}$$

**Variables**

- $X_{ik}$: contracts traded
- $A_{ik}$: absolute trade size

# Example Absolute Value Linearization

Suppose:

$$X_{ik} = -7$$

Constraints imply:

$$A_{ik} \geq -7$$
$$A_{ik} \geq 7$$

Therefore:

$$A_{ik} \geq 7$$

Since the objective minimizes $A_{ik}$:

$$A_{ik} = 7 = |X_{ik}|$$

**Variables**

- $X_{ik}$: contracts traded
- $A_{ik}$: absolute trade size

# Challenge 5: Limiting Unique Options

Question:

How do we limit the number of distinct SPX options used?

# Solution: Big-M Formulation

Define:

$$U_i \in \{0, 1\}$$

Activation constraint:

$$U_i \geq \frac{1}{M} \sum_{k \in K} A_{ik}$$

Limit total options:

$$\sum_{i \in I} U_i \leq N$$

**Variables**

- $U_i$: whether option $i$ is used
- $A_{ik}$: trade size
- $N$: max number of options

# Challenge 6: Choosing M

Question:

How large should $M$ be?

# Solution: Big-M Construction

$$M = \max(|\ell^X|, |u^X|) \cdot |K| + 1$$

Reasoning:

$$A_{ik} \leq \max(|\ell^X|, |u^X|)$$
$$\sum_{k \in K} A_{ik} \leq \max(|\ell^X|, |u^X|) \cdot |K|$$

Thus:

$$\frac{1}{M} \sum_{k \in K} A_{ik} < 1$$

**Variables**

- $M$: sufficiently large scaling constant
- $\ell^X$, $u^X$: trade bounds
- $|K|$: number of funds

# Objective Function

$$\min \sum_{i \in I} \sum_{k \in K} A_{ik}$$

Interpretation:

- Minimize total gross SPX option trading
- Encourage sparse, simple hedge implementations

**Variables**

- $A_{ik}$: absolute trade size
- $i$: option index
- $k$: fund index

# Business Interpretation

The optimizer seeks:

- Robust downside protection
- Controlled upside exposure
- Minimal trading footprint
- Small number of SPX instruments